cess. Misery is the only result of its evolution, and, when at last the misery culminates in human consciousness, there is but one way of deliverance open, the suicide of the universe, to be achieved, strangely enough, through moral conduct and the universal will of humanity. This is Hartmann's solution for the evils of life, "cosmic suicide," "humanity hurling back into nothing the world process." Could anything be imagined more fantastic or bizarre? Its jaunty affectation is wholly different from the gloom of Schopenhauer, which has at least the merit of reality, and gives a certain dignity to his pessimistic theories.

The question remains: What is the future of pessimism? In order to answer it, we have but to compare its doctrines with the nature of the human will and of human activity. We have but to see how it contradicts itself, how it distorts and misinterprets the purest and highest of all spiritual forces—love, the power of self-sacrifice. Standing half way between realism and positivism, pessimism merely proves how impossible it is to banish from thought that Divine Idea of the Absolute which has been the strength and consolation of man throughout the ages. As a philosophic system, pessimism may from time to time exert a momentary influence in the world's history. But it will not endure; the duties of each day, useful and necessary activity, will dissipate its evil dreams and save humanity. The question is not simply one of happiness and misery, but of right and wrong. Philosophy must observe this distinction, or, failing to satisfy our highest needs and aspirations, it will lead to spiritual sterility and spiritual death.

ON THE SYMBOLIC SYSTEM OF LAMBERT.

BY JOSEPH JASTROW.

Joh. Heinr. Lambert (1728–1777) was a logician of no mean rank, as his influence on German thought has shown; it was he whom Kant called "der unvergleichlicher Mann." His first logical work was "Neues Organon oder Gedanken über die Erforschung und Bezeichnung des Wahren und dessen Unterscheidung vom

Irrthum und Schein" (Leipzig, 1764); this was followed by an article in the "Nova Acta Eruditorum," in 1765. His later writings are, "Anlage zur Architectonik" (1771); "Logische und philosophische Abhandlungen" and "Deutscher gelehrter Briefwechsel," both published posthumously in 1781. I have only had access to the "Neues Organon," and the object of this note is to review his system as there set forth. I have availed myself of Mr. Venn's copious notes (Symbolic Logic); Hamilton (Lectures on Logic) and Thomson (Laws of Thought) also have some references.

The opening chapter of the second volume contains Lambert's general idea of a symbolic system, under the title "Von der symbolischen Erkenntniss überhaupt." The origin of symbolism is in language. The different languages are so many symbolic systems. Each word must be a symbol of something; if this is not so, there results, not knowledge, but a word-cram. The ideal system is one in which the signs of the concepts and the things perfectly correspond, so that one can be put for the other. For this it is necessary that the relations involved by the things should also be involved in the signs. Music-notes, the points of the compass, the signs of the zodiac, those of astronomy ($^{\circ}\ '\ ''$), of chemistry, etc., etc., are examples of symbolic methods. Arithmetic, however, is a more remarkable one: "For it is no small thing to express by means of ten figures—or in the Leibnitzian 'Dyadik, of two—all possible numbers, and to perform all calculations, and that too, in such a mechanical way that it can be done by machines, such as Pascal, Leibnitz, Ludolf, and others have invented. In this we reduce the theory of the things to that of the signs; and we are so used to this that the numbers soon come to be regarded as nothing but signs, while in fact they are concepts of relations" (§ 34). Algebra is the most perfect system, because its own theory is a symbolic art. For "if you reduce a problem from another science to an algebraic one, you can abstract from the former, and the solution of the algebraic problem will also be that of the other. There are two kinds of symbols in algebra, the letters of the quantities and the operation symbols for expressing relations; the one is an "Allgemeine Zeichenkunst," the other "Verbindungskunst der Zeichen." The introduction of the latter, says Mr. Venn, marks the real turning-point in symbolic logic. Lambert conceives the object of this art to be the determination of possible combinations,

the degree of their validity, of their mutual relations, and the laws of their interchange, etc. (§ 41).

Symbols are either natural or artificial; smoke as a sign of fire, the symptoms of a disease are natural signs; the tolling of bells an artificial one. Ordinary symbols are more or less arbitrary; mere imitation may be the natural element. Even in algebraic equations we can introduce the conception of a pair of scales with equal weights on its arms.[1] What the signs do not of themselves indicate, the doctrine about them must show; and the signs will be the more complete the closer they follow the fact. Signs are more scientific, however, the better they mark the conditions, etc., determined by the things themselves which the sign marks. Thus, in algebra, the problem, when completely solved, tells not only what the answer is, but all the circumstances, whether more answers than one, what data are superfluous or wanting, and, if the solution is impossible, tells where it begins to be so, and so on.

Let this suffice to show that Lambert had worked out a theory of symbolism, both interesting and valuable, and that his system of logical notation, being comprehended, like that of Leibnitz, under this more general symbolism, could not fail to be related to that other important symbolic system, mathematics. Lambert's logical system is extremely complete and original. He recognizes the importance of the natural element (for on this he bases the distinction of the four figures of syllogism); he lays stress on the importance of induction and of the theory of probabilities, and has himself worked out the elementary departments of each. Speaking in general of symbolic logic, Mr. Venn says: "To my thinking, he and Boole stand quite supreme in this subject in the way of originality; and, if the latter had knowingly built on the foundation laid by his predecessor instead of beginning anew for himself, it would be hard to say which of the two had actually done the most" (p. xxxii, *op. cit.*). Hamilton's verdict is in singular opposition. After enumerating eight objections (with one exception these objections stated are to be either (1), misconceptions of Lambert; or (2), matters of opinion in which a great deal can be said for Lambert; or (3), Hamiltonian peculiarities), he

[1] Is it not just such conceptions and illustrations which are so valuable for educational purposes? To reduce to terms of sight what is expressed in terms of thought is the germ of this symbolic procedure.

words the ninth thus: "Lambert—but it is needless to proceed. What has already been said shows that Lambert's scheme of linear notation is, in its parts, a failure, being only a corruption of the good and a blundering and incongruous jumble of the natural and conventional. The only marvel is, how so able a mathematician should have propounded two such worthless mathematical methods. But Lambert's geometrical is worse even than [his] algebraic[1] notation" (p. 668, *op. cit.* New York, 1809). To Hamilton "mathematical" and "worthless" seem almost identical in the system of Logic. Mr. Venn's criticism of Hamilton's scheme is this (p. 432): "It has been described (by himself) as 'easy, simple, compendious, all-sufficient, consistent, manifest, precise, complete, the corresponding antithetic adjectives being freely expended in the description of the schemes of those who had gone before him. To my thinking, it does not deserve the rank as a diagrammatic scheme at all, though he does class it with the others as 'geometric;' but it is purely symbolical. What was aimed at in the methods above described was something that should explain itself, as in the circles of Euler, or need but a hint of explanation, as in the lines of Lambert. But there is clearly nothing in the two ends of a wedge to suggest subjects and predicates, or in a colon and comma to suggest distribution and non-distribution." Every diagrammatic scheme must be somewhat symbolic; it is all a question of degree and of naturalness, and in both these respects Hamilton's goes beyond the boundaries of legitimacy.

Lambert's first notation was the linear, and of that I will give some account. Every notion has some extension. Let a series of dots denote individuals, and the line will denote the notion (vol. i, p. 110). The relative length of the lines is not entirely arbitrary; the real length is. If our knowledge were more perfect, these lengths would be more definite. This perfection is, however, ideal. To this Mr. Venn objects (*op. cit.*, p. 430). "Thus Lambert certainly seems to maintain that in strictness we must suppose each line to bear to any other the due proportionate length assigned by the extension of the terms." But Lambert's "in strictness" means in an ideal world where we had perfect knowledge. That for us the lengths of these lines are entirely arbitrary, he dis-

[1] It should be said that Mr. Venn treats almost entirely of the algebraic notation.

tinctly says. The "due proportionate length" has also no reference to feet and inches. It means that, if we compare A and C with B, B is a sort of a standard for A and C, and if A is found to contain B, and B to contain C, their relative lengths are determined. Mr. Venn adds: "In the latter part of the "Neues Organon"—where he is dealing with questions of probability, and the numerically, or rather proportionately, definite syllogism—the length of the lines which represent the extent of the concepts becomes very important. So little was he prepared to regard the diagram as referring solely to the purely logical considerations of presence and absence, of class characteristics, of inclusion and exclusion of classes by one another." I cannot find any diagrams in the chapter on Probability that present this feature; if he had used diagrams, he would have done what Mr. Venn objects to. But the proportionately definite syllogism and probability are not purely logical considerations, and what the length of the lines would denote would not be a logical but a mathematical conception. To this I can see no objection.

Lambert[1] expresses "All A is B" by $\begin{array}{l}\text{B————————b}\\ \quad\text{A—————a}\end{array}$, or $\begin{array}{l}\text{....B————————b......}\\ \quad\text{A—————a}\end{array}$, in the former case B is definite, in the latter not. We can write the converse thus, $\begin{array}{l}\qquad\text{A—————A}\\ \text{...B————————b.....}\end{array}$, showing that it is undetermined whether "all B is A," but that surely "some B is A." But we need no separate diagram for the converse; we drop the distinction of subject and predicate and read the diagram in any order. The metaphor here is that of one concept being conceived under another. Of course, this is little more than a play upon words; Lambert regards it just as he does the idea of a pair of scales for the equation. "No A is B": A—————a B————————b, or ...A—————... ...B——b... if indeterminate; and the converse is evidently true, No B is A. Some A is B. $\begin{array}{l}\text{B————————b}\\ \text{.....\ A\ .....}\end{array}$, in which it is undetermined whether All A is B or All B is A. It may be written $\begin{array}{l}\text{....B—————b....}\\ \text{.....A—————a.....}\end{array}$, and sometimes $\begin{array}{l}\qquad\text{B———b}\\ \text{...A————————a...}\end{array}$, the last pointing to a Univ.

[1] Vol. i, p. 112, *sqq.*

Aff. as the converse. "Some A is not B": $\begin{array}{l}\quad B\text{———————}b \\ \ldots\ldots A\ldots\ldots\end{array}$.

It may be $\begin{array}{l}\quad B\text{———————}b \\ \ldots A\text{—}a\ldots\end{array}$ or $\begin{array}{l}\quad B\ldots\ldots\ldots \\ A\text{———}a\end{array}$. Here I must dispute a statement of Mr. Venn (*op. cit.*, p. 431). Speaking of the employment of dotted lines to express indeterminateness, he says: But when he comes to extend this to particular propositions, his use of dotted lines ceases to be consistent or even, to me, intelligible. One would have expected him to write "some A is B" thus, $\begin{array}{l}B\text{————}\ldots\ldots \\ \;A\text{———}\ldots\ldots\end{array}$, for, by different filling in of the lines, we could cover the case of there being "B which is not A," and so forth. But he does draw it $\begin{array}{l}\;B\text{——} \\ A\ldots\ldots\ldots\end{array}$, which might consistently be interpreted to cover the case of "no A is B," as well as suggesting the possibility of there being "no A at all." Lambert does give the form that one would expect him to give, and he does not give the other; for he expressly says that, by putting the letter A under B, we are sure of having at least one individual A which is B. You have no right to put the A outside of B. These forms, says Lambert, show not only the necessary differences between two propositions, but also how far the converse is true, and how far true when conditions are changed, and how determinate the conclusion is. There is no necessary order of the lines nor of subject and predicate. If a genus A has three species, B, C, D, we would write $\begin{array}{l}A\text{—————————————————}a \\ \;B\text{———}b\; C\text{————}c\; D\text{———}d\end{array}$ where the lengths of the lines are arbitrary. Disjunctives cannot be expressed at all, since they tell nothing positive. A is either B or C. This only says "No B is C." Conjunctives can be written: A is B and C $\begin{array}{l}\ldots C\text{————}c\ldots\ldots \\ \ldots B\text{———}b\ldots \\ \quad\; A\text{—}a\end{array}$, which shows that A is B as well as C, that some B is C and some C, B. The copulative A, as well as B is C, can be written in two parts, thus: $\begin{array}{l}C\text{————}c \\ \;A\text{——}a\end{array}$ $\begin{array}{l}C\text{————}c \\ \;B\text{——}b\end{array}$. If we write both under one line, we do not know whether to put them beside or above another. $\begin{array}{l}C\text{——————}c \\ \;B\text{———}b \\ \;A\text{——}a\end{array}$ or

C————c
B——b or C——————c, where all, some, and no A is B.
. . . . A . . . . A—a B—b

If we know this we can select our form.

. . . . P————P . . . .
Barbara will be M———M, which gives—(1) Some
S—S

M is S; (2) some P is M; (3) some P is S; (4) all S is P. The problem is, whether, by drawing the lines representing the premises, you condition the lines representing (not one, but) all the conclusions. He then develops all the moods and figures according to this scheme; but it would be tedious to follow him there. There are several objections to this scheme; one, Lambert himself has pointed out, viz., that it cannot represent disjunctives. How to represent disjunctives diagrammatically, I do not know. Let us approach the question this way: every diagrammatic system is intimately connected with the material view of logic. If every logical expression stands for a state of affairs, why should that state of affairs not be capable of being diagrammatically expressed, or, so to speak, painted? The answer is evident. A is either B or C does represent a state of affairs, but one in which the subjective element is not entirely eliminated. It is the ball in the air which is going to fall on one of two places, *I* don't know which. In point of fact, objective causes have settled on what spot the ball is going to fall, but I am in doubt; and doubt is subjective. Lambert expressed his reason thus: that, after putting B and C aside of each other, you have nothing but a conditional to tell you whether to put A under B or under C. Another difficulty is to make "Some A is B" and "Some A is not B" perfectly distinct. This Lambert does by the position of the letter A and the dots as marks of indeterminateness, which latter is symbolic rather than diagrammatic. There is, however, no objection to this, *per se*. But it leads to a plurality of forms, according to the different ways of filling out the dots, which is confusing. Hamilton[1] accuses him of making one diagram answer for two syllogisms. Thus, he says, Datisi, Disamis, Bocardo are the same. The only difference between Disamis and Datisi is in the order of the premises; and this Lambert properly expresses by the different positions

[1] P. 670.

of B and C. That Bocardo is the same is one of Hamilton's mistakes. Lambert gives Disamis

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B————b
M———m
...C......
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, and Bocardo

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B ————— b
M———m
  C........
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. Where the important difference is, that, in the latter case, it is determinate in one direction, and in that direction we find the M and the B, that is not C. It cannot be too strongly maintained that Hamilton's criticisms are very unjust. As I admitted before, these different positions of the dots are confusing. The chief value of the scheme is its completeness, and its strict adherence to the rule that the lines representing the premise determine the conclusion. Of more value still are the general principles of which this scheme is an outcome; besides, if his algebraical method is valuable, this borrows some of its worth, since it is in this that the germs of the former are to be found.

It would seem that, since Lambert gives up the distinction of subject and predicate, he ought also to neglect the figures, and formally he does. But he claims that the distinction of figures is a natural one; they have different uses, and each has its dictum. For the first figure: *Dictum de omni et Nullo.* What is true of all A, is true of every A. For the second figure: *Dictum de Diverso.* Things which are different are not attributes of each other. For the third figure: *Dictum de Exemplo.* When we find things A which are B, in that case some A are B. For the fourth figure: *Dictum de Reciproco.* I. If No M is B, then no B is this or that M. II. If C is [or is not] this or that B, in that case some B are [or are not] C.[1]

Let us follow out another part of Lambert's Logic which is intimately connected with his later doctrines. Starting from the fact that from two particulars no conclusion follows, he notes that, if the "some" is the same "some" in both, we get a conclusion; for then we really have, not an indefinite some A, but a new term, mA. On this principle we treat singulars as universals, because

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[1] I should add Thomson's note (p. 173, *op. cit.*). "But Mr. Mill is in error, shared by Buhle (Geschichte, vi, 543), and Troxler (Logik, ii, p. 62), in thinking that Lambert invented these dicta. More than a century earlier Keckermann saw that each Figure had its own law and its peculiar use, and stated them as accurately, if less concisely than Lambert. Keckermann, however, ignored the 4th Figure, and Lambert's explanation of that may be new.

they are perfectly definite, e. g., "The earth is inhabited," "the earth is a planet ∴ (at least) one planet is inhabited." He completes his scheme by considering the effect of one premise being false. In his chapter on problems there is much of interest. If all A are B then also mA are B, and all mA will be mB. All triangles are figures; all right-angled triangles are right-angled figures. But can you get A is B from mA is mB, as well as mA is mB from A is B (a step analogous to multiplication in algebra)? You can get mA is B, but whether m can be dropped from the subject is another question. If our language were strictly logical, we could. We sometimes conclude that all mA is m, neglecting the principal notion B. If you have mA is nA, you can get mA is n, but it is uncertain whether n belongs to A or m or mA. Then follows an interesting study of the method of generalizing problems. His problems are solved mostly by the means of the identity A is A, and the principle that mA is m and mA is A; they are mostly theoretical, bearing on the relation between data and quæsita. I will close this very brief sketch by summing up his chapter on Probability. A is $\frac{3}{4}$B means that A has $\frac{3}{4}$ of the marks of B. $\frac{3}{4}$A is B means that $\frac{3}{4}$ of the A's are B's. A$\frac{3}{4}$ is B means that the probability is $\frac{3}{4}$ that A is B. A simple case is $\left.\begin{array}{r}\frac{3}{4}\text{A are B}\\ \text{C is A}\\ \therefore\ \text{C}\frac{3}{4}\text{ is B}\end{array}\right\}$, which shows where the probability comes in, and how much it is. If we have the second premise, all C is A, the conclusion will be, all C$\frac{3}{4}$ are B; if some C is A, then the conclusion will be indefinite; if the some is definite, we have $\left.\begin{array}{r}\frac{3}{4}\text{A are B}\\ \frac{2}{3}\text{C are A}\\ \frac{2}{3}\text{C }\frac{3}{4}\text{ are B}\end{array}\right\}$. In all these cases, the probability of the conclusion arises from the major premise. We will now consider the case in which it arises from the minor premise. M N P Q are marks of B, then B is M N P Q. Now C is M N P, then probably C is B. If the marks M N P Q = A, and M N P = $\frac{2}{3}$A, then we have All A is B, C is $\frac{2}{3}$A ∴ C$\frac{2}{3}$ is B. The next form is obtained by compounding these two: $\left.\begin{array}{r}\frac{3}{4}\text{A are B}\\ \text{C is }\frac{2}{3}\text{A}\\ \therefore\ \text{C}\frac{1}{2}\text{ is B}\end{array}\right\}$. If we make the major negative, we would have $\frac{1}{4}$A are not B. C is $\frac{2}{3}$A ∴ C$\frac{1}{6}$ is not B. Then $\frac{1}{2}$C are B, $\frac{1}{6}$ are not B, and the other $\frac{1}{3}$ are undetermined. We see all along how carefully he

distinguishes between probability of intension and of extension. The intension probability becomes the formula for induction. Let *a* be the affirmative, *e* the negative, and *u* the undetermined part of the probability, then we would have a case such as this: $(\frac{2}{3}a+\frac{1}{4}e+\frac{1}{12}u)A$ are B; C is $(\frac{3}{5}a+\frac{2}{5}u)A$ ∴ $C(\frac{2}{5}a+\frac{3}{20}e+\frac{9}{20}u)$ is B; which means that, of 20 cases, C will be B 8 times, will not be B 3 times, and will remain doubtful 9 times; or in any one case there are 8 chances of finding the Ca B, 3 of not finding it aB, and 9 chances of its remaining doubtful. The multiplication is algebraic; remembering that anything containing *u* belongs under *u*, and that *ae* belongs under *e*. $\frac{4}{5}$ $A\frac{1}{3}$ is $\frac{3}{8}$B. $A\frac{1}{10}$ is B, because, when interpreted, they represent the same state of affairs. This sort of probable reasoning is not confined to two premises by any means. In general, if mA are B, nA are C. Where $n>m$, then (1) $(n-m)A$ will be C but not B; (2) if $m+n>1$, then $(m+n-1)A$ are B and C, or if $m+n<1$, then $(1-n-m)A$ are neither B nor C. He develops this method, using figures and words, and applies it to the calculation of the probability of testimony, and so on.

If we view these doctrines in the light of recent logical ones, they lose a great deal of their value, but little of their interest. The doctrine that a particular cannot be obtained from a universal will invalidate many of his diagrams; and other results of more general methods render any such treatment superfluous. His merit consists in having so clearly grasped the principles on which all such investigation depends. But Lambert did not stop here. The "Neues Organon" was only his first work; and, according to Mr. Venn, all his best symbolic speculations are to be found in the later works, particularly the "Logische Abhandlungen." I will conclude by giving Mr. Venn's summary of Lambert's speculations as derived from the later works ("Symbolik Logik," p. xxxii, *sqq.*). "Summarily stated, then, Lambert had got as far as this. He fully realized that the four algebraic operations of addition, subtraction, multiplication, and division, have each an analogue in logic; that they may there be respectively termed aggregation [Zusammensetzung], separation [Absonderung], determination [Bestimmung], and abstraction [Abstraction], and be symbolized by +, —, ×, ÷.[1] He also perceived the inverse nature of the

[1] By mistake + is printed instead of ÷.

second and fourth as compared with the first and third;[1] and no one could state more clearly that we must not confound the mathematical with the logical signification.[2] He enunciates with perfect clearness the principal logical laws, such as the commutative, the distributive, and the associative,[3] and (under restrictions to be presently noticed) the special law[4] AA = A. He develops simple logical expressions precisely as Boole does,[5] though without assigning any generalized formulæ for the purpose.

"He fully understood that the distinctive merit of such a system was to be found in its capacity of grappling with highly complicated terms and propositions; and he accordingly applies it to examples which, however simple they may seem to a modern symbolist, represent a very great advance beyond the syllogism.[6] Moreover, in this spirit of generalization, he proposed an ingenious system of notation, of a 1 and 0 description, for the 2^n combinations which may be yielded by the introduction of n class terms or attributes.[7] Hypothetical proportions he interpreted and

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[1] "Die Operationen + und — sind einander entgegengesetzt und sie leiden einerlei Verwechselungen wie in der Algeber." ("Logische Abhandl.," ii, p. 62.)

[2] "Wir haben die Beweise der Zeichnungsart kurz angezeigt, die Zeichen selbst aus der Algeber genommen, und nur ihre Bedeutung allgemeiner gemacht." (*Ibid.*, i, 37.)

[3] "Da man in vielen Sprachen das Adjectivum vor- und nachsetzen kann, so ist es auch einerlei ob man nR oder Rn setzt." (*Ibid.* i, p. 150.) "Da es in der Zeichenkunst einerlei ist ob man a + b oder b + a setzt." (*Ibid.*, i, p. 33.) "Will man aber setzen (m + n) A, so ist dieses = mA + nA. Es sei m = n + p + q. Und A = B + C + D + E. So hat man mA = (n + p + q) (B + C + D + E). . . ."

[4] "Man kann zu einem Begriffe nicht Merkmale hinzusetzen die er schon hat . . . weil man sonst sagen könnte *eisernes Eisen.*" (*Ibid.*, ii, p. 133.) The reason why he did not admit this law universally was (as presently noticed), that he endeavored to make his formulæ cover *relations* as well as common logical predications. This comes out clearly in the following passage: "Wenn der Begriff = α ist, $\alpha\gamma$ das Geschlecht, $\alpha\gamma^n$ ein höheres Geschlecht, $\alpha\delta$ der Unterschied, $\alpha\delta^n$ ein höherer Unterschied, $\alpha\gamma + \alpha\delta = \alpha$, die Erklärung $(\alpha\gamma + \alpha\delta)^n$ oder $\alpha\,(\gamma + \delta)^n$, eine höhere Erklärung," i. e., α being a true logical class-term $\alpha^n = \alpha$; but γ, being a relative term, γ^n does not $= \gamma$. (*Ibid.*, p. 133.)

[5] His formula is $a = ax + a \mid x$ (where $a \mid x$ means a not-x, viz., our $a\bar{x}$). He also has $x + y = 2xy + x \mid y + y \mid x$; just as Boole develops the expression.

[6] Take, for instance, the following: F :: H = S :: (P + G) :: V (A + C + Se) as expressive of "Die Glükseligkeit des Menschen besteht in der Empfindung des Besitzes und Genusses der Volkommenheiten des innerlichen und äusserlichen Zustandes." The sign :: here denotes a relation. (*Ibid.*, i, p. 56.)

[7] His scheme is this: Let 1 represent the presence and 0 the absence of the attribute. If we keep the order in which the terms stand in our expression unaltered, 10101 and 10111 will take the place of what we might indicate by $x\bar{y}z\bar{w}v$ and $x\bar{y}zwv$. He

represented precisely as we should.[1] Still more noteworthy is the fact, that in one passage, at least, he recognized that the inverse process marked by division is an *indeterminate* one.[2] These are the main truths of this kind which Lambert had seized. Whatever the defects and limitations in their expression, they represent a very remarkable advance on anything known to have been done before him. Where he mainly went astray was, I think, in the following respects: Though he realized very clearly that logical division is the inverse of multiplication, he failed to observe the indefinite character commonly assumed by inverse operations; that is, he failed to observe it except in certain special cases, as just pointed out.

"He regarded the inverse as being merely the *putting back* a thing, so to say, where it was before,[3] and accordingly omitted altogether that surplus indefinite term yielded by logical division, and which is so characteristic of Boole's treatment. Probably no logician before Boole (with the very doubtful exception of H. Grassmann) ever conceived a hint of this, as not many after him seem to have understood or appreciated it. As a consequence of this, Lambert too freely uses mathematical rules which are not justifiable in logic. For instance, from AB = CD he assumes that we may conclude A : C = D : B.

then compares the extent to which various complex terms thus agree with each other or differ. He also employs the slightly more convenient notation of letters and their negation, thus: ABC, AB0, A00, and so on, to stand for our ABC, $AB\bar{C}$, $A\bar{B}\bar{C}$. (*Ibid.*, ii, 134.) Of course there are great imperfections in such a scheme."*

[1] "Die allgemeinste Formel der hypothetischen Sätze ist diese, wenn A ein B ist, so ist es C. Diese Formel kann allezeit mit den folgenden verwechselt werden; alles A so B ist ist C. Nun ist alles A so B ist = AB. Folglich, alles AB ist C. Daher die Zeichnung AB > C oder AB = mC." (*Ibid.* i, 128.)

[2] "Wenn $x\gamma = \alpha\gamma$, so ist $x = \alpha\gamma\gamma^{-1} = \alpha\frac{\gamma}{\gamma}$. Aber deswegen nicht allezeit $x = \alpha$; sondern nur in einem einzigen Falle, weil x und α zwei verschiedene Arten von dem Geschlecht $x\gamma$ oder $\alpha\gamma$ sein können. Wenn aber $x\gamma = \alpha\gamma$ nicht weiter bestimmt wird, so kann man unter andern auch $x = \alpha$ setzen." (*Ibid.*, i, 9.) (As this expressly refers to relative terms only, it is not at variance with the note below, at p. 80.)

[3] "Auch ist klar dass man sich dabei Operationen muss gedenken können, wodurch die veränderte Sache in den vorigen Stand könnte hergestellt werden. Diese Wiederherstellung giebt demnoch den Begriff der reciproken Operationen, dergleichen im Calcul + und −, × und ÷." (*Ibid.*, ii, p. 50.)

* Judging from the extract in Johns Hopkins University Circular, No. 10, p. 131, this seems to be similar to Dr. Franklin's scheme.

"Another point that misled Lambert was the belief that his rules and definitions would cover the case of *relative* terms.[1] . . . I think it a mistake to endeavor thus to introduce relative terms; but, if we do so, we must clearly reject the law that $x^2 = x$, in the case of such terms.

"In thus realizing what Lambert had achieved, the reader must remember that he by no means stood alone. Two of his friends or correspondents—Plonquet and Holland—are worthy coadjutors; and such logical writings as they have left behind are full of interesting suggestions of a similar kind. . . . These men all took their impulse from Leibnitz and Wolf."

GIORDANO BRUNO.

TRANSLATED FROM HEGEL'S "HISTORY OF PHILOSOPHY," BY EDWIN D. MEAD.

Giordano Bruno was one of those restless, troubled, seething spirits, like Cardanus, Campanella, and Vanini, who appeared in Italy in the sixteenth century. He utterly rejected all the old catholic reliance on authority, and fell back boldly upon his own reason. His memory has been revived in these later times by Jacobi, who appended an extract from one of Bruno's works to his "Letters on Spinoza." Jacobi drew special attention to him by his assertion that the sum and substance of his doctrine was the same as Spinoza's "One and All," or pantheism—a comparison which lifts Bruno to a position really above that to which he is justly entitled.

Bruno's life was perhaps a steadier and quieter life than that of Cardanus; but he, too, had no fixed abiding place in the world. He was born at Nola, near Naples, some time in the sixteenth century, the exact year not being known.[2] He became a Dominican monk, but quickly had occasion to speak out upon the gross

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[1] "Unter den Begriffen M = A : B, kommen einige vor, die sehr allgemein sind. Dahin rechnen wir die Begriffe; Ursache, Wirkung, Mittel, Absicht, Grund, Art, Gattung." (Architectonik, i, 82.)

[2] About 1548.—TR.